Adiabatic computation with Josephson junction circuits

Jens Siewert

DMFCI, Università degli Studi di Catania, Italy, and
Institut für Theoretische Physik, University of Regensburg, Germany

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it is possible

- to design systems with tailor-made Hamiltonians by using superconducting circuit technology

- to perform adiabatic manipulations on these circuits

Outline

1. Superconducting qubits

- ingredients
- flavors of superconducting qubits

2. Simple examples for adiabatic manipulations

- adiabatic sweeping for a single charge qubit
- Landau-Zener tunneling
- adiabatic passage (STIRAP)

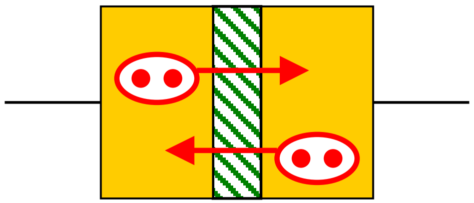
3. Couplings for charge qubits

- tunable couplings
- towards true applications

Superconducting qubits

Elements of superconducting nanocircuits

Josephson tunnel junction

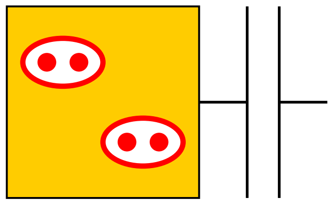


- phase difference φ
- critical current I_C

- Josephson energy

$$E_J = (\hbar/2e)\, I_C$$

small superconducting islands



- charge $Q = 2e\, n$
- capacitance $C \sim 1\ \mathrm{fF}$

- charging energy

$$E_C = (2e)^2 / 2C$$

Hamiltonian -- macroscopic quantum dynamics

general form of Hamilton operator for Josephson junction circuits:

$$H = E_C\,(\,n - n_x\,)^2 - E_J(\Phi)\cos(\,\varphi - \alpha(\Phi))$$

dynamic variable
island charge

control parameter
offset charge

dynamic variable
junction phase

control parameter
external flux bias

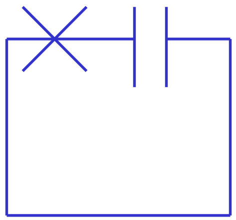
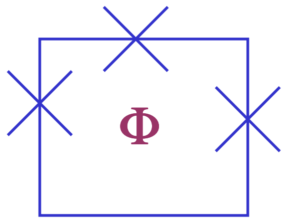
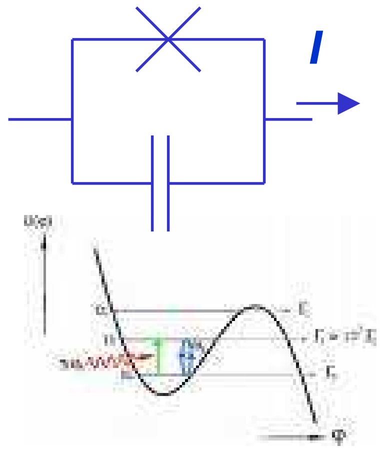
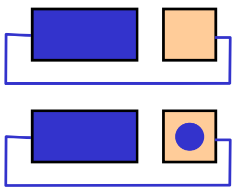
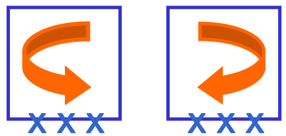
required for quantum dynamics

$$[\,n\,,\,\varphi\,] = -\,i$$

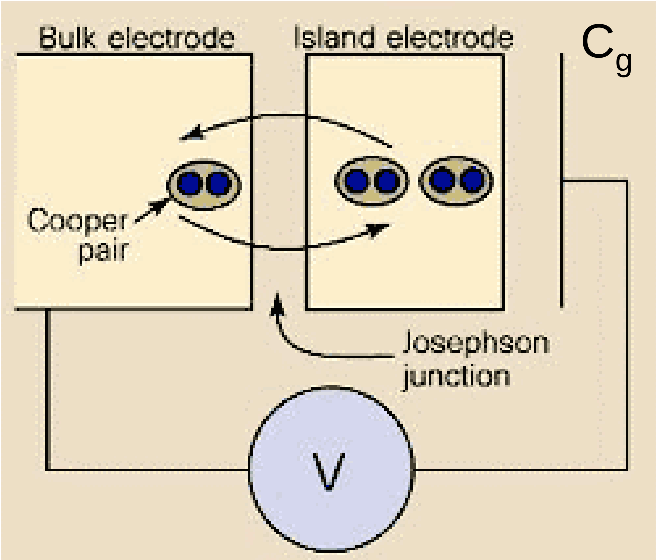
Anderson 1964
Ambegaokar, Eckern, Schön 1982

→ diagonalize Hamiltonian, choose two lowest eigenstates as the two-level system

Flavors of superconducting qubits

	charge qubit	flux qubit	phase qubit
	E_C >> E_J	**E_C << E_J**	**E_C << E_J**
circuit			
computational states	$\vert 0 \rangle$ $\vert 1 \rangle$ 		
	island charge states	**persistent current states**	**phase eigenstates in potential well**
	Nakamura et al Tsukuba Delsing et al Gothenburg Wallraff et al Yale + ETH [Esteve et al Saclay]	Mooij et al Delft Clarke et al Berkeley	Martinis et al UCSB

Cooper-pair box – the charge qubit



E_C >> E_J

$n_x = C_g V$ – induced gate charge

Büttiker 1987
Bouchiat et al 1997

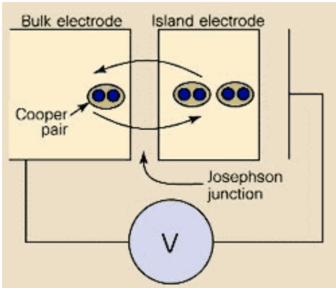
$$H = E_c\,(n_x - 1/2)\,\sigma_z - E_J / 2\;\sigma_x$$

$E_C \sim 400\ \mu\text{ eV}$
$E_J \sim 40\ \mu\text{ eV}$

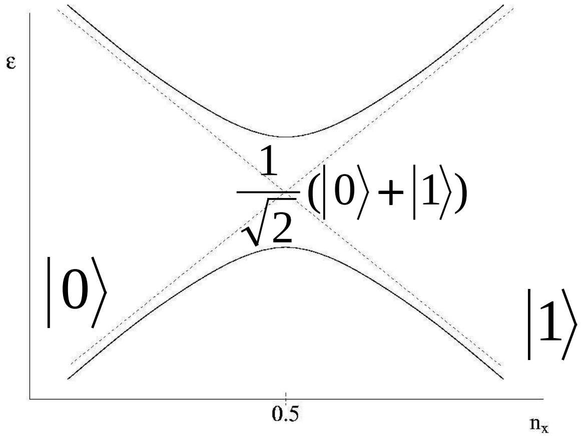
measurement → charge measurement of island charge ... can be done

Simple adiabatic manipulations

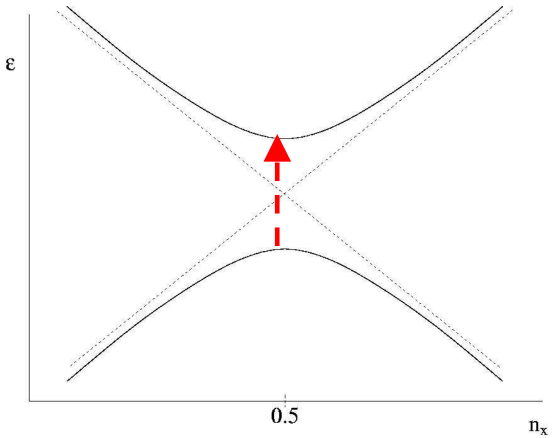
Adiabaticity condition – Landau-Zener tunneling



sweep gate voltage V = 0 ... 2e/C_g



← $C_g V$ →



Hakonen et al 2006

$$p_{LZ} = \exp(-\pi E_J^2 / 4\hbar E_C \dot{n}_x)$$

probability for Landau-Zener transition

adiabaticity for $\tau >> \hbar / E_J$

~ nanoseconds

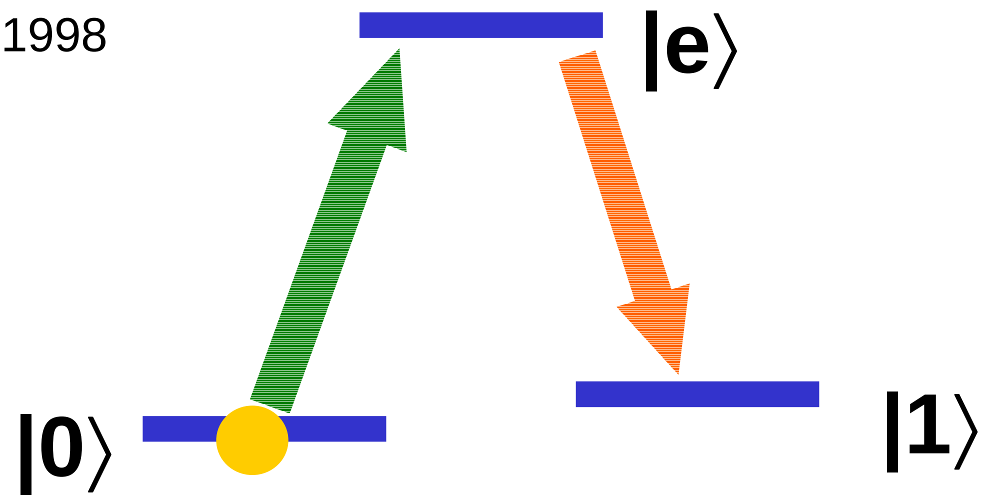
Adiabatic passage in three-state system

more elaborate example:

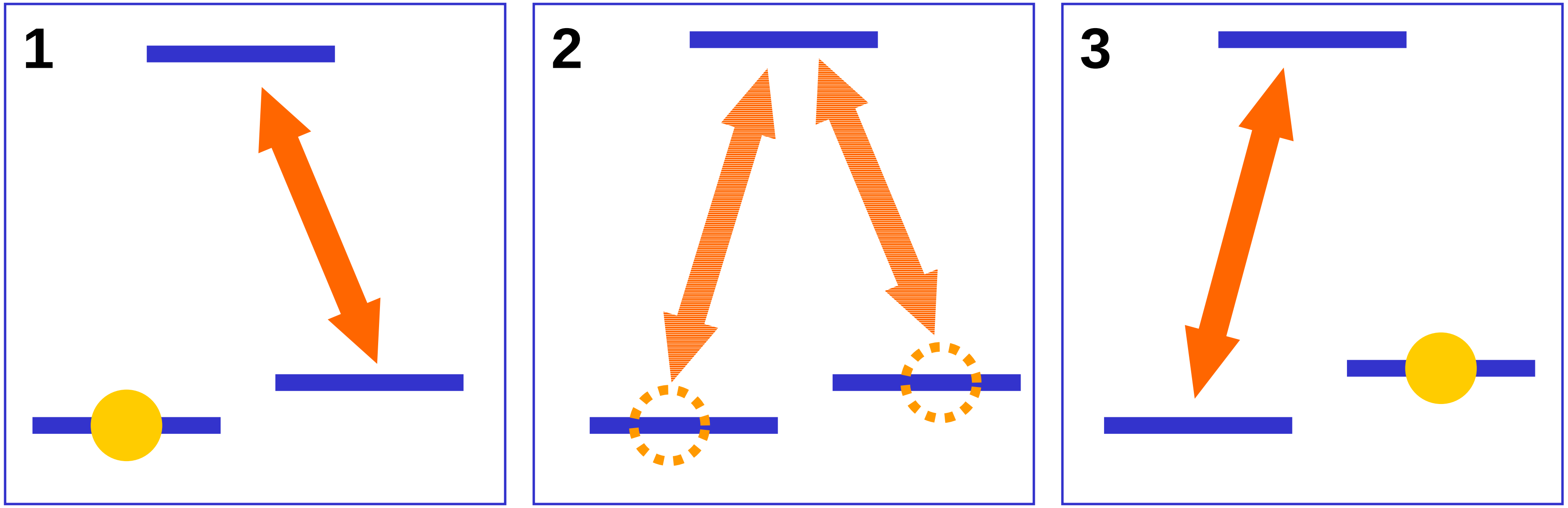
analogue of STIRAP technique in quantum optics

"Λ configuration"

Bergmann, Theuer, Shore 1998

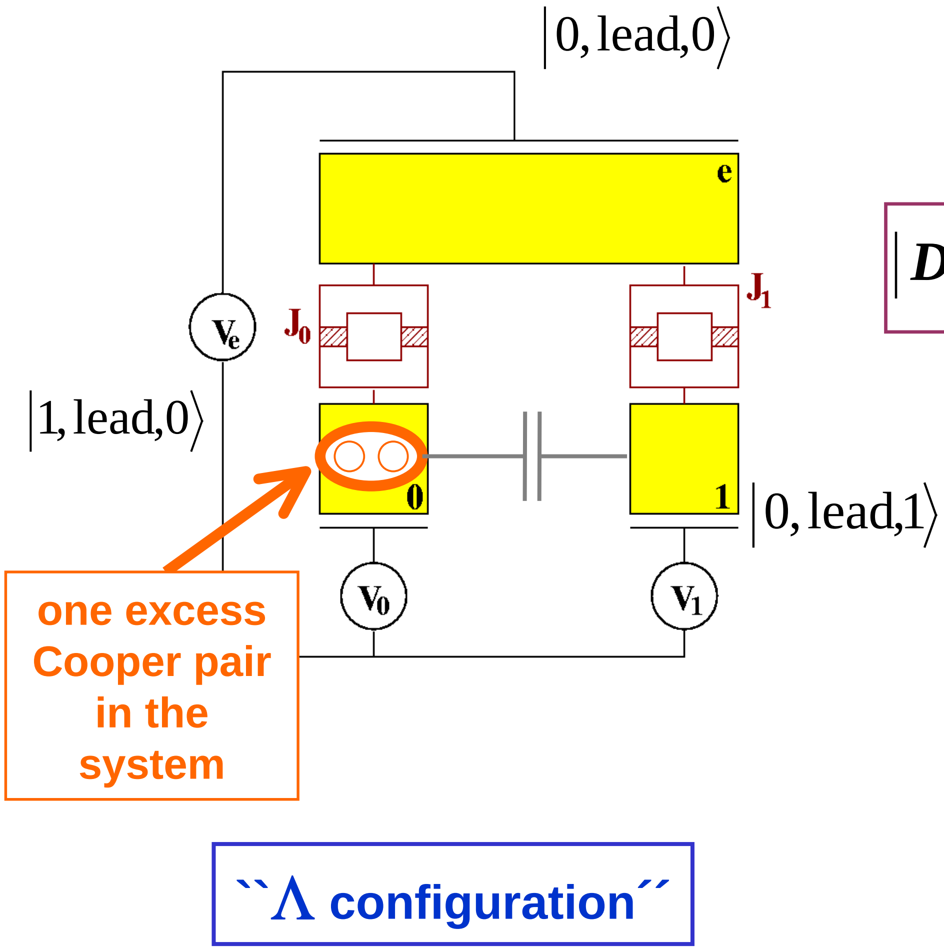


coherent population transfer → ‘counterintuitive scheme’



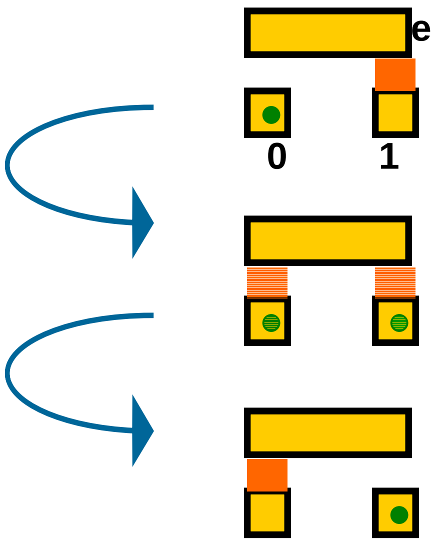
adiabatic switching of couplings

Adiabatic passage with two-island device



"dark state"

$$|D\rangle(t) = (J_1(t)|1,\text{lead},0\rangle - J_0(t)|0,\text{lead},1\rangle)$$



Siewert, Falci, Brandes 2006

Couplings for charge qubits

Couplings for charge qubits

required for true adiabatic computation:

→ coupling between qubits

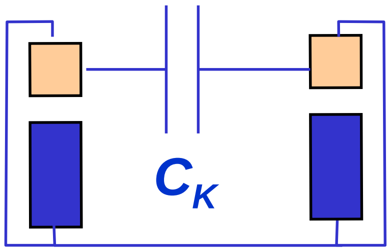
$$H_{\text{coupl}} = \sum J_{ij}^{ab} \sigma_i^a \sigma_j^b$$

→ many possibilities, e.g.,

capacitive	$\sigma^z \sigma^z$	
inductive	$\sigma^y \sigma^y$	Shnirman et al 1997
Josephson junctions	$\sigma^+ \sigma^-$	Siewert et al 2000
inductive	$\sigma^x \sigma^x$	You et al 2002
capacitive (tunable)	$\sigma^z \sigma^z$	Averin, Bruder 2003
current bias	$\sigma^x \sigma^x$	Lantz et al 2004
cavity-mediated	$\sigma^+ \sigma^-$	Blais et al 2004

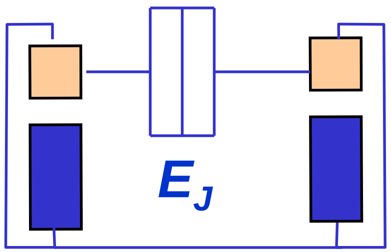
Couplings for charge qubits

capacitive coupling



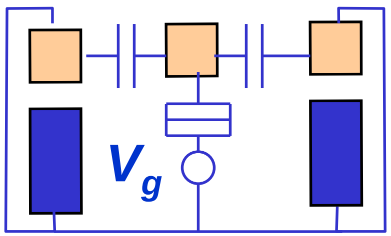
$$H_{\text{coupl}} \propto C_K \sigma_1^z \sigma_2^z$$

coupling by Josephson junctions



$$H_{\text{coupl}} \propto E_J(\Phi)(\sigma_1^+ \sigma_2^- + \sigma_1^- \sigma_2^+)$$

tunable capacitive coupling



$$H_{\text{coupl}} \propto \varepsilon(V_g)\sigma_1^z \sigma_2^z$$

Simple example for adiabatic computation

find the ground state of a spin-glass system
(version of MAXCUT problem)

$$H = \sum \varepsilon_i s_i + \sum J_{ij} s_i s_j, \quad s_i = \pm 1$$

random (but fixed) coupling parameters

Simple example for adiabatic computation

has been considered theoretically for 3 flux qubits (Il'ichev et al PRA 2005)

$$H = \sum \varepsilon_i \sigma_i^z + \sum J_{ij} \sigma_i^z \sigma_j^z$$

- prepare ground state with $J_{ij} = 0$
- (slowly) switch on couplings
- read out final state

Conclusions

- superconducting nanocircuits provide flexibility to design tailor-made Hamiltonians

- quantum dynamics of macroscopic objects (!)

- adiabatic manipulations are feasible (many interesting protocols: LZ, adiabatic passage, Abelian + non-Abelian holonomies)

- realization of adiabatic quantum computation may be a different story