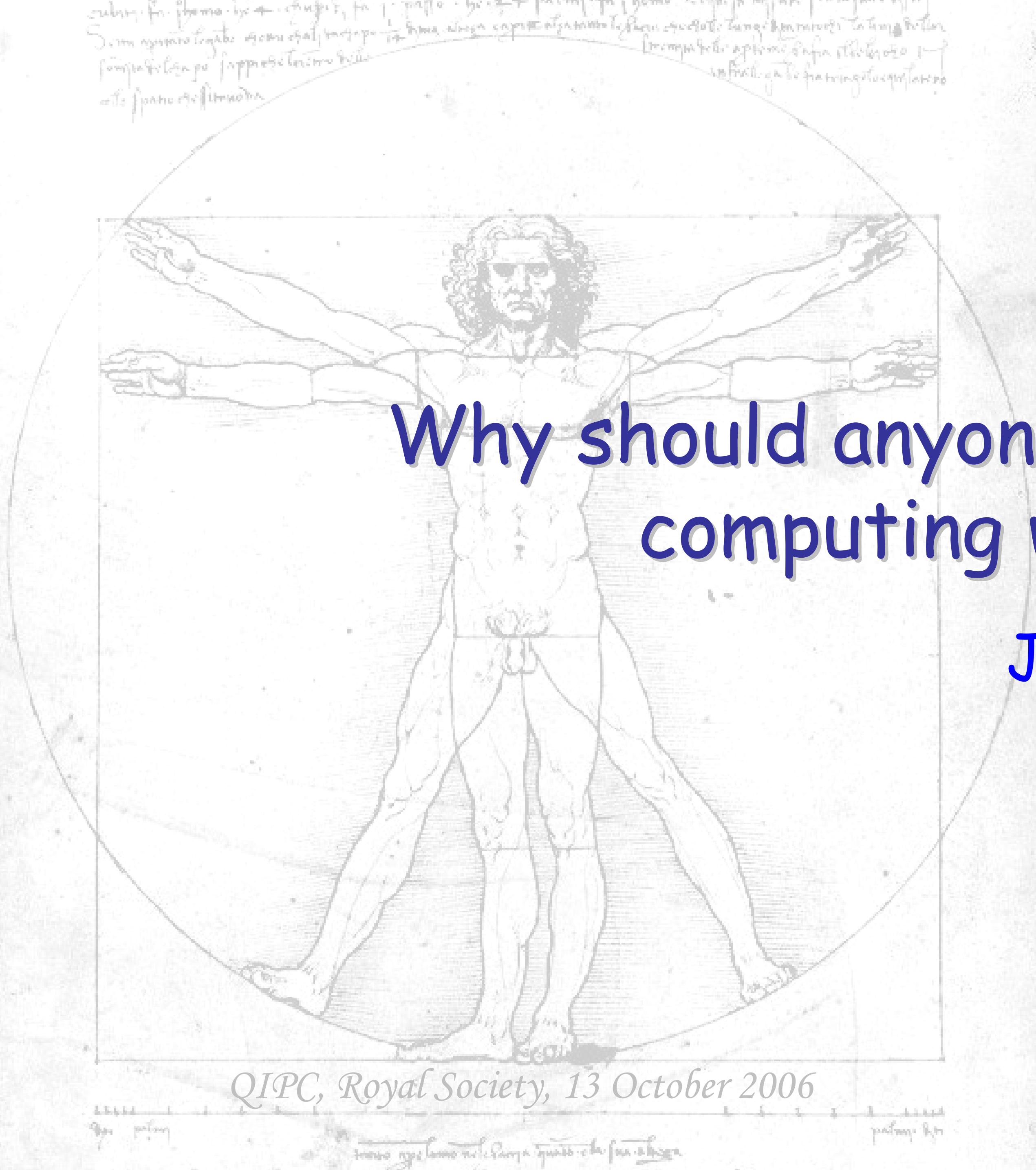


Why should anyone care about computing with anyons?

Jiannis K. Pachos

QIPC, Royal Society, 13 October 2006

Overview

- Quantum Computation: the quest for
  - neat quantum evolutions
  - new quantum algorithms

- Topological Quantum Systems: QHE, 2D lattices
  - Support anyons
  - Topological Degeneracy
  - Charge Fractionalization
  - Effective gauge theories

[Wen; Bais; Zoller; Ortiz;…]

Overview

- Quantum Information Encoding and Processing
  - –Anyons carry charges (similar to spin)
  - –Charges change by braiding anyons
- Jones polynomials: a topological invariant of knots
  - –Exponentially hard to evaluate classically
  - –Polynomially easy to approximate with QC
- Intrinsic resilience to decoherence and control errors:
  - –Non-local encoding of information: resilience against local decoherence

[Kitaev (1997); Freedman; Preskill; Rasetti;…]

for your great kindness in the matter of the names respecting which I applied to you; but I hoped to have met you last Saturday at Kensington and therefore delayed expressing my obligations.

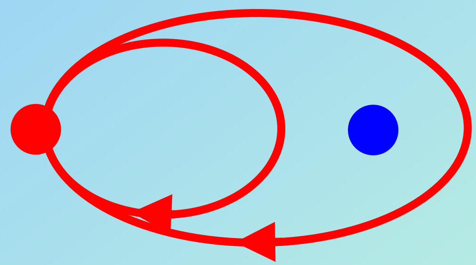
I have taken your advice and the names used are anode cathode anions cations and ions the last I shall have but little occasion for. I had some hot objections made to them here and found myself very much in the condition of the man with his son and Ass who tried to please every body; but when

Letter from Faraday to Whewell (1834)

Topological Quantum Systems: Anyons

- Two dimensional systems
- Dynamically trivial (H=0), interested only in statistics
- They can support anyons (non-trivial statistics)

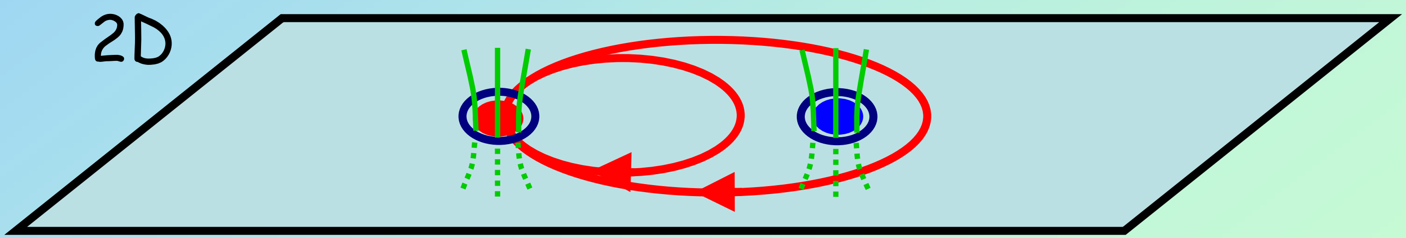
3D



Bosons $|\Psi\rangle \rightarrow |\Psi\rangle$

Fermions $|\Psi\rangle \rightarrow e^{i2\pi}|\Psi\rangle$

2D

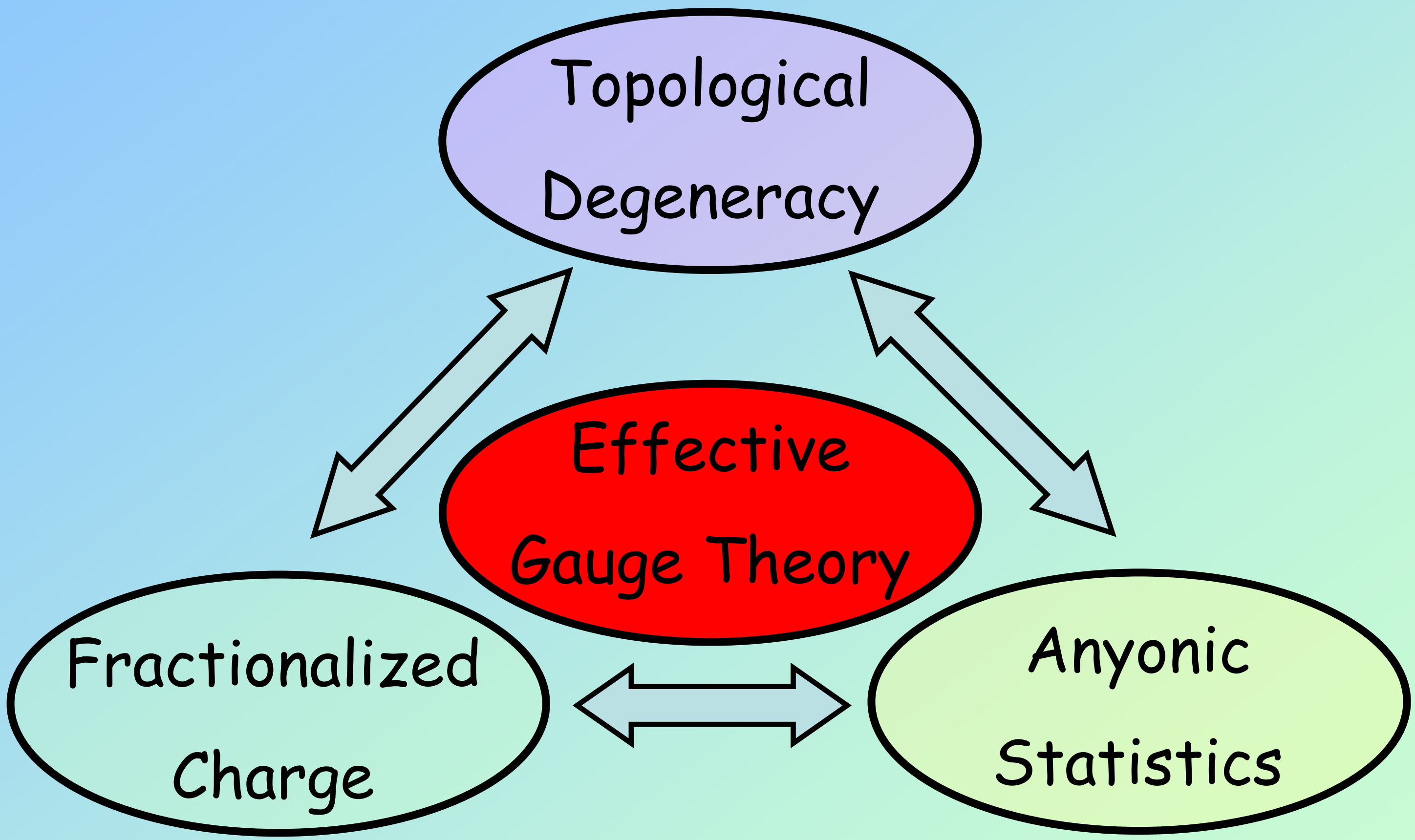


$|\Psi\rangle \rightarrow e^{i2\varphi}|\Psi\rangle$

$|\Psi\rangle \rightarrow U|\Psi\rangle$

Anyons

Topological Quantum Systems

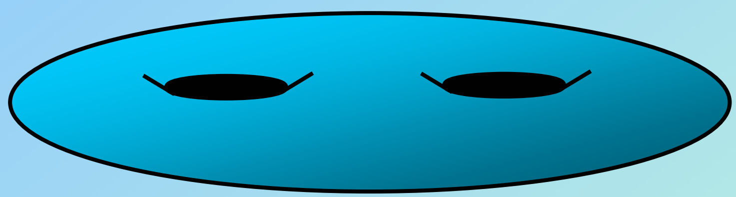


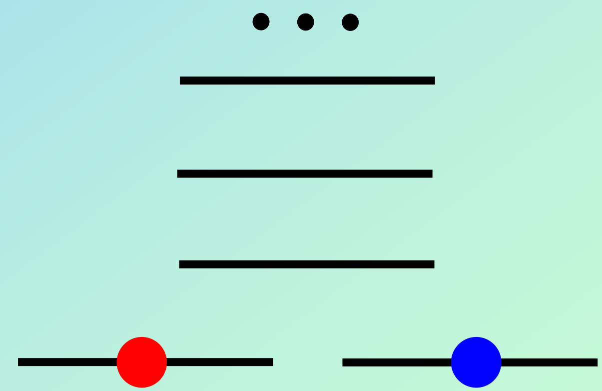
Topological Degeneracy

System with **degenerate ground states** where:

- The degeneracy is protected by topology (genus)
- Degenerate states are not locally distinguishable

g=2





$\Rightarrow$ **encode information in degenerate subspace**

Resilience to local decoherence (local magnetic fields, geometric deformations,…)

Topological Degeneracy

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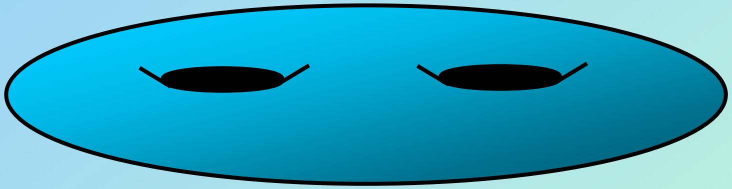
g=2 

Index Theorem: [Atiyah, Singer (1963)]

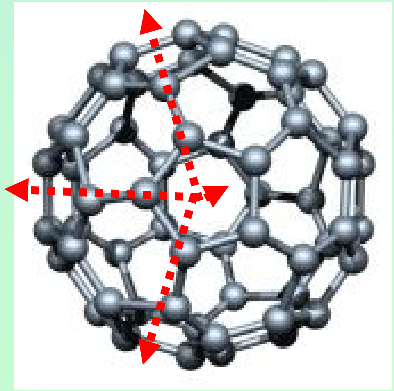
$$n_+ - n_- = \frac{1}{4\pi}\iint B d^2 x$$

:integer, n=n(g)

n fermionic zero energy modes

=> 2^n degenerate ground states

E.g. fullerenes, nanotubes… [JKP, Stone (2006)]



Anyonic Properties

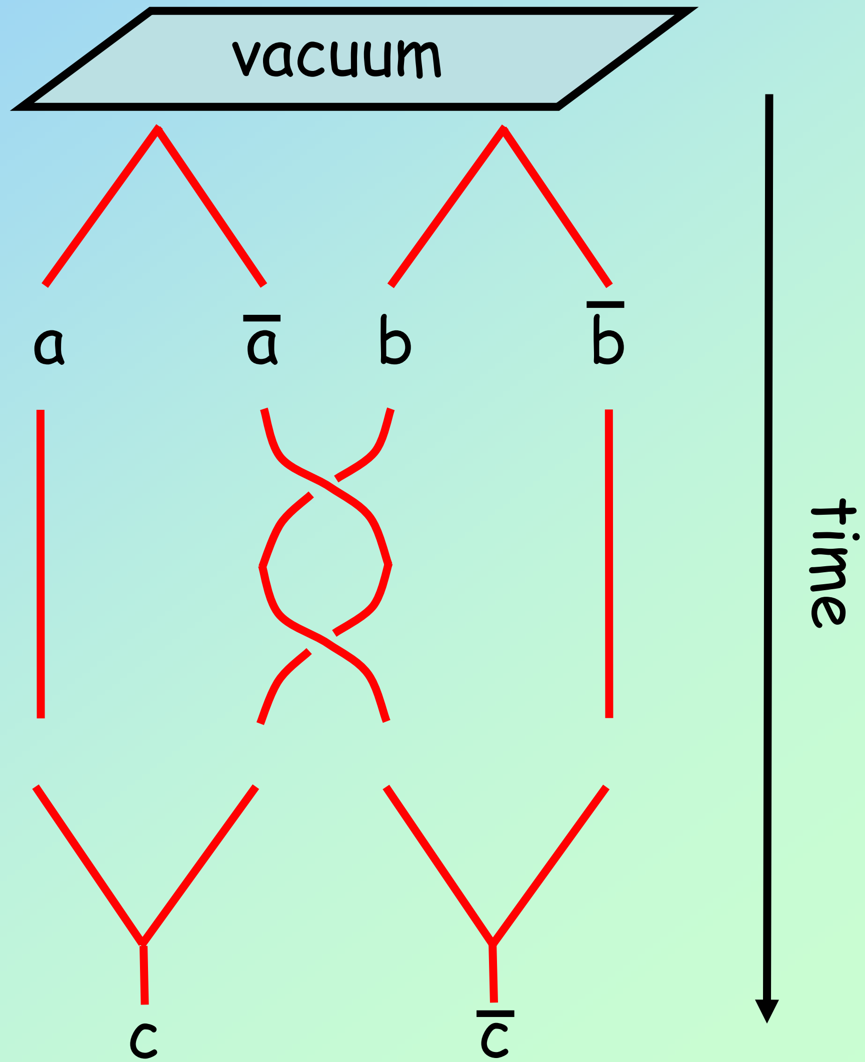
- Assume we can:
  - **Create** identifiable anyons

    e.g. measure them by interference experiments

    [Slingerland et. al (2006)]
  - **Braid** anyons

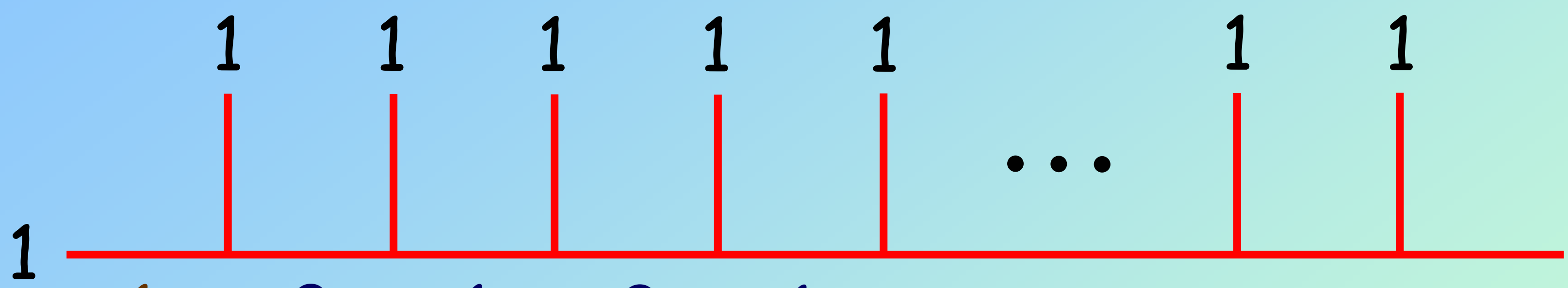
    trap and move them
  - **Fuse** anyons

    e.g. axb=c+d+...



Fibonacci Anyons

Consider n anyons with charge 0 or 1 with the fusion properties: 0x0=0, 0x1=1, 1x1=0+1



1 0 1 0 1

1 0 1 0

1 1 1

0 0 . . .

1 1

1

0

1

$d_1=1$

$d_2=2$

$d_3=3$

$d_4=5$

$d_5=8$

…

Fibonacci sequence!

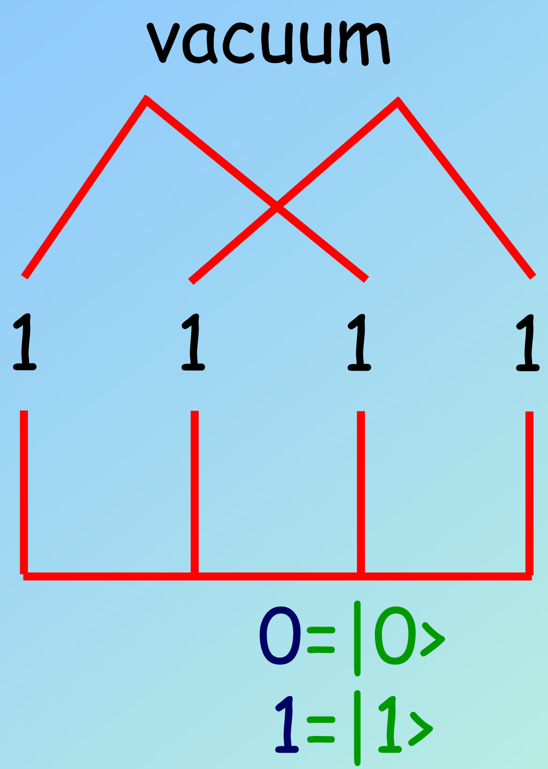
$$d_n \propto \phi^n$$

$$\phi = (1+\sqrt{5})/2$$

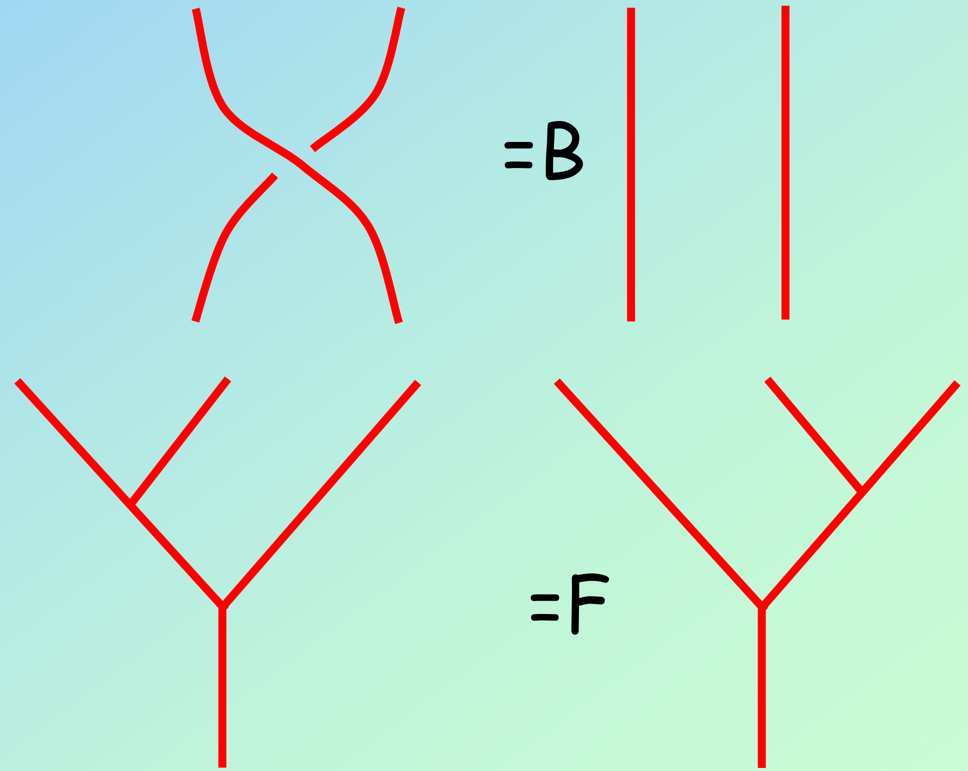
Golden mean

Fibonacci Anyons and QC

Qubit encoding:

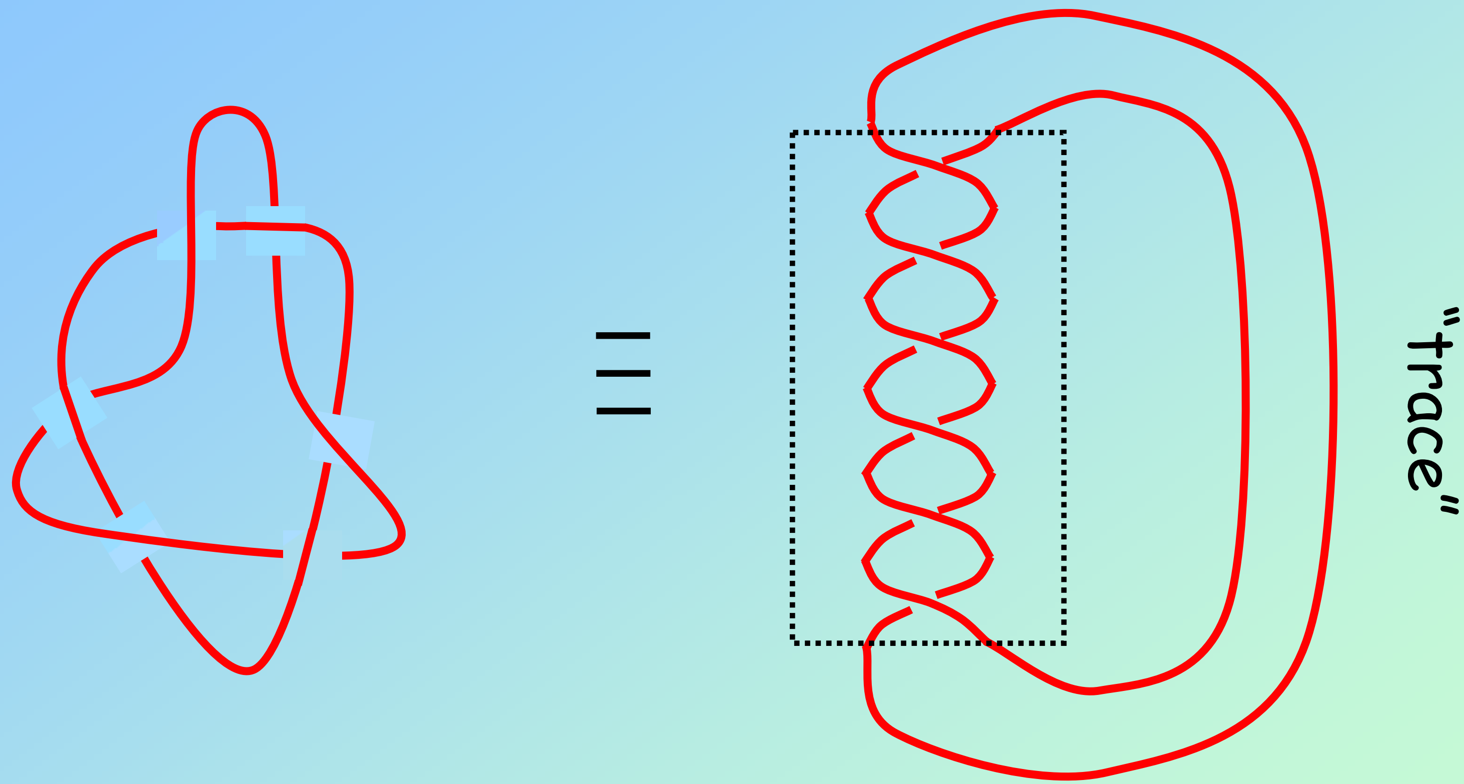


Evolving a qubit:



Unitaries B and F are dense in SU(2).
Extends to $SU(d_n)$ when n anyons are employed.

Approximating Jones Polynomials

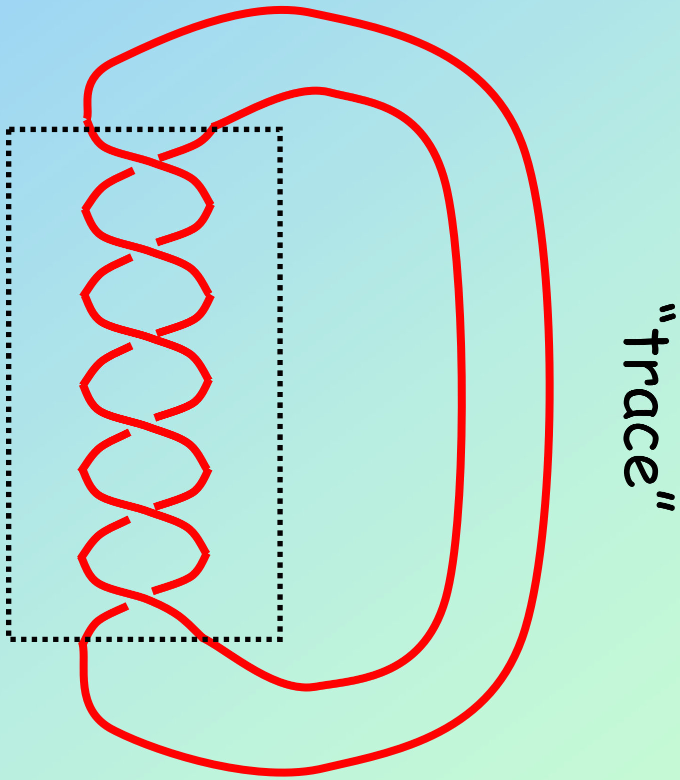


Knots (and links) are equivalent to braids with a "trace".

[Markov, Alexander theorems]

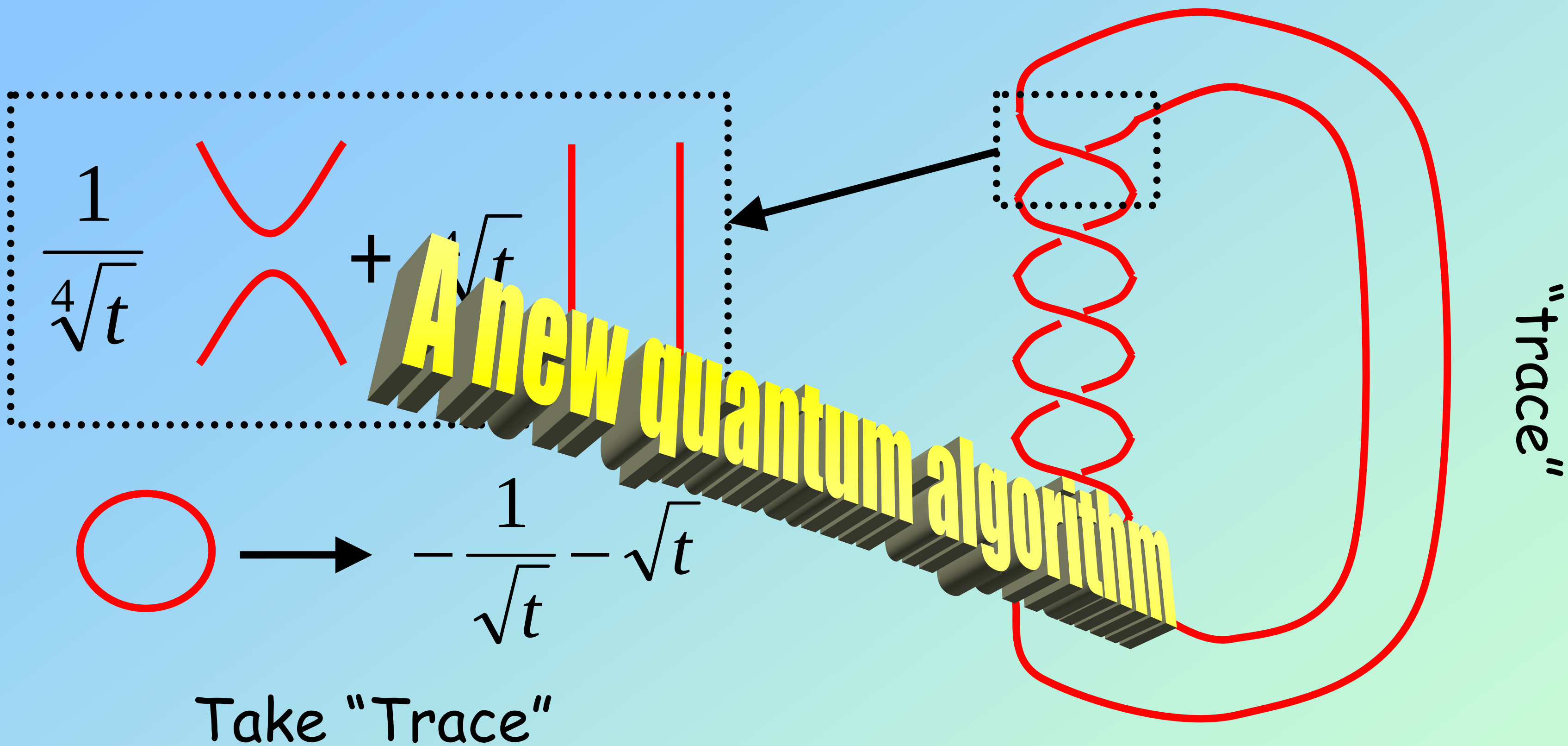
Approximating Jones Polynomials

$\equiv$



Is it possible to check if **two knots are equivalent or not**?
The **Jones polynomial** is a topological invariant:
it is different for non-equivalent knots.
[Jones (1985)]
Exponentially hard to evaluate classically.
Applications: DNA reconstruction, statistical physics...

Approximating Jones Polynomials



With quantum computers it is **polynomially** easy to approximate: *Simulate the knot with anyonic braiding and make appropriate measurement.*

[Freedman, Kitaev, Wang (2002); Aharonov, Jones, Landau (2005)]

Resilience to Errors

- **Abandon** the idea of separate subsystems for **qubits**. Encode info in **macroscopic degree of freedom**.
- To measure the total charge of two anyons you need to fuse them. Direct observation of each anyon does not reveal their charge.

=> local decoherence (environment "measures") does not destroy information. **Information is stored non-locally**.

- The **unitary transformations** resulting from braiding are virtually **errorless** as they depend only on the topological characteristics of the anyonic trajectories.

We should $\sim 10^{-4}$ we get $\sim 10^{-30}$

Conclusions

- Topological Quantum Computation promises to overcome the problem of decoherence and errors in the most direct way.

- There is lots of work to be done to make anyons work for us.

- Is it worth it?

Aesthetics says YES!